

# A METHOD FOR FETAL HEART RATE EXTRACTION BASED ON TIME-FREQUENCY ANALYSIS

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## Abstract

*A three-stage method for fetal heart rate extraction, from abdominal ECG recordings, is proposed. In the first stage the maternal R-peaks and fiducial points (QRS onset and offset) are detected, using time-frequency analysis, and the maternal QRS complexes are eliminated. The second stage locates the positions of the candidate fetal R-peaks, using complex wavelets and pattern matching theory techniques. In the third stage, the fetal R-peaks that overlap with the maternal QRS complexes are found. The method is validated using a dataset of 4 long duration recordings and the obtained results indicate high detection ability of the method (96% accuracy).*

## 1. Introduction

The fetal electrocardiogram (FECG) can be derived from the abdominal ECG (AECG) and be used for the extraction of fetal heart rate (FHR), which is a marker for the cardiac condition of the fetus [1]. Various research efforts have been carried out in the area of FECG and FHR extraction, including subtraction of an averaged pattern [2], matched filtering [3], adaptive filtering [4-6], orthogonal basis functions [7], fractals [8], FIR [9], dynamic neural networks [10], temporal structure [11], fuzzy logic [12], frequency tracking [13], polynomial networks [14], and real-time signal

processing [15]. The wavelet transform (WT) is another approach that has been proposed for FECGs processing. Several techniques for noise removal and detection of fetal waveforms have been used, involving Gabor-8 wavelets and Lipschitz exponent's theory [16], bi-orthogonal quadratic spline wavelet, modulus maxima theory [17] and complex continuous wavelets [18].

The extraction of FECG from the complex signal (mother and fetus) can be reframed in a more efficient manner using Blind Source Separation (BSS) methods [19]. BSS methods are used to extract unobserved signals (called sources), which are assumed to be statistically independent, from a known mixture of these signals. The BSS methods are divided into two major groups, the ones that use second-order statistics, performing Principal Component Analysis (PCA) [20, 21] or Singular Value Decomposition [20,22], and those that take advantage of the higher-order statistical information contained in the available data, performing Independent Component Analysis (ICA) [5, 20]. Some proposed ICA-based techniques are the JADE algorithm [23] and the FastICA algorithm [24]. Combining wavelet analysis and BSS methods has also been proposed [25,26]. The main disadvantage of the BSS-based methods is that they require a large number of recorded ECG leads, including thoracic leads, in order to have adequate results.

In this paper, a three-stage method for the automated detection of fetal heart beats and the FHR extraction is presented. The method is based on the analysis of leads of the AECG signal. In the first stage,

the AECG is analysed using smoothed pseudo Wigner-Ville distribution (SPWVD). The areas of high energy concentration are detected and they are used to detect the maternal R-peaks and fiducial points (QRS onset and offset). The maternal fiducial points are used to eliminate the maternal QRS (MQRS) complexes from the AECG. In the second stage, complex wavelets are used in order to detect the candidate fetal R-peaks. False cases (artefacts) are included but not fetal R-peaks overlapping with the MQRSs, because they are eliminated in the previous stage. Acceptance or rejection of the candidate fetal R-peaks is materialized utilising pattern matching techniques, based on a set of fetal QRS complex (FQRS) patterns, proposed by medical experts. The detection of the overlapped fetal R-peaks with the MQRSs is accomplished in the third stage, using a histogram-based technique. The detected fetal R-peaks are used to extract the FHR. The flowchart of the proposed method is shown in Fig. 1.

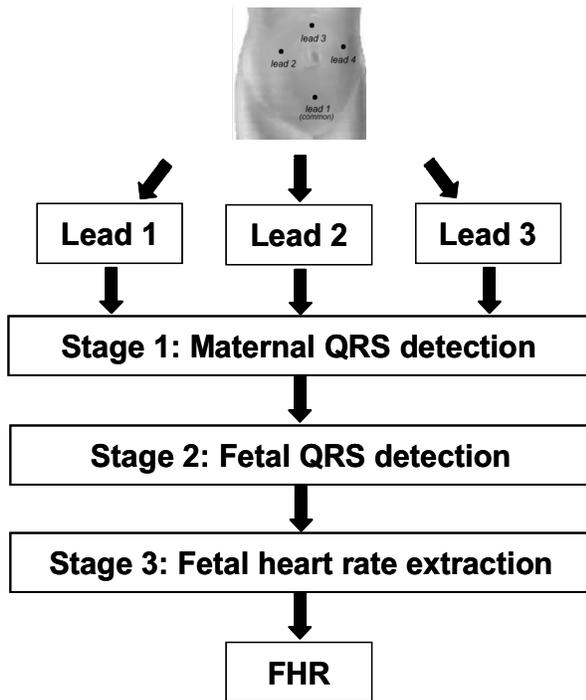


Figure 1. The proposed 3-stage method

## 2. Materials and Methods

The initial signal is acquired using electrodes (leads) placed on the mother's abdomen. Each of the stages of the three-stage method (Fig. 1) is described below.

### 2.1. Maternal QRS elimination

The input leads are averaged and the DC of the average signal is removed (Eq. 1). The final signal is named AECG:

$$AECG(x) = \sum_{i=1}^M lead_i(x) - \frac{1}{N} \sum_{x=1}^N \sum_{i=1}^M lead_i(x), \quad (1)$$

where  $M$  is the number of the available leads and  $N$  is the length (number of samples) of each lead. The smoothed pseudo WVD (SPWVD), defined as:

$$SPWVD_x(t, \omega) = \int_{-\infty}^{+\infty} h(\tau) \left( \int_{-\infty}^{+\infty} g(s-t) x\left(s + \frac{\tau}{2}\right) x^*\left(s + \frac{\tau}{2}\right) e^{-j2\omega\tau} d\tau \right), \quad (2)$$

where  $x(t)$  is the signal and  $h(\cdot)$ ,  $g(\cdot)$  are window functions, is applied to the AECG. The time window is a Hamming 64-point length window, while the frequency one is a Hamming 32-point length window. The function is integrated with respect to frequency, in order to marginalize the signal's energy distribution over time:

$$E(t) = \left( \int_{-\infty}^{+\infty} SPWVD(t, \omega) d\omega \right)^2. \quad (3)$$

Using the  $E(t)$  of the AECG, the maternal R-peaks are detected as follows: for a time instant  $t_i$ , if  $E(t_i) > \theta$  then  $t_i$  is considered as a candidate maternal R-peak. The threshold  $\theta$  is adaptive to each AECG record:  $\theta = 0.9 \frac{1}{N} \sum_{i=1}^N E(t_i)$ . Two medical rules are applied to include or reject a candidate maternal R-peak: for two consecutive candidate maternal R-peaks located at  $t_i$  and  $t_j$ ,

**R1:** if  $t_j - t_i < 0.2s$  then the candidate maternal R-peak corresponding to the minimum  $E(\cdot)$  value is rejected (two R-peaks cannot exist closer than 0.2s).

**R2:** if  $t_j - t_i > 2s$  then the time instant corresponding to the maximum  $E(\cdot)$  value between the candidate maternal R-peaks, is considered as a candidate maternal R-peak (a time interval greater than 2s does not exist without an R-wave).

The above rules are applied recursively until no candidate maternal R-peak is rejected or a new candidate maternal R-peak is included, and the remaining maternal R-peaks are considered as the actual maternal R-peaks. For each of them, the maternal Q wave starting point (MQRS onset) and S wave end (MQRS offset) are detected, using the  $E(\cdot)$  of the AECG as follows: a window of 101 samples is centred at the  $i^{\text{th}}$  maternal R-peak ( $t_i$ ) and the sub windows  $[t_{i-50}, t_i)$  and  $(t_i, t_{i+50}]$  are searched for the Q wave start and the S wave end, respectively. The local minima of  $E(\cdot)$  are found in both sub windows. The first local minima before the  $t_i$  point defines the end of the Q wave ( $t_{Q_i}^{\text{end}}$ ), while the second local minima after the  $t_i$  point ( $t_{Q_i}$ ) defines the time interval  $[t_{Q_i}, t_{Q_i}^{\text{end}}]$  where the starting point of the Q wave ( $t_{Q_i}^{\text{start}}$ ) is searched. The  $t_{Q_i}^{\text{start}}$  is defined as:  $\int_{t_{Q_i}^{\text{start}}}^{t_{Q_i}^{\text{end}}} E(t) < 0.99 \int_{t_{Q_i}}^{t_{Q_i}^{\text{end}}} E(t)$ . The same procedure is followed for the identification of the  $t_{S_i}^{\text{start}}$  and  $t_{S_i}^{\text{end}}$  points. The  $i^{\text{th}}$  MQRS is eliminated, setting the values of the AECG signal between the  $t_{Q_i}^{\text{start}}$  and  $t_{S_i}^{\text{end}}$  points to zero. Fig. 2a presents the initial AECG signal and Fig. 2b presents the AECG signal with the MQRS eliminated, named eECG.

## 2.2. Fetal QRS detection

In the second stage, initially, the eECG is analyzed using complex continuous wavelet transform (CCWT) [27-29] and then the modulus of the coefficients is used to find the candidate fetal R-peaks using pattern

matching. The complex nature of the wavelets provides further improvement in signal processing, compared to real-valued wavelet analysis. Using the square modulus of CCWT the maxima and minima can be found. The complex frequency B-spline wavelet [30], is used in our algorithm, defined as:

$$\psi(x) = \sqrt{f_b} \left( \sin c \left( \frac{f_c x}{m} \right) \right)^m e^{2\pi i f_c x}, \quad (4)$$

where  $m$  is an integer order parameter,  $f_b$  a bandwidth parameter and  $f_c$  the wavelet's centre frequency. A first order wavelet ( $m = 1$ ) is used, for computational reduction and, after comparative analysis, the parameters are chosen as  $f_b = 1$  and  $f_c = 0.5$ . The fetal R-peak detection is accomplished using the modulus of the CCWT coefficients (mCCWT). The mCCWT signal is searched for local maxima using peak detection based on adaptively changing threshold value. A window is centred on each peak and, if there is a higher peak in this window, then the initial peak is rejected. The remaining peaks are considered as candidate fetal R-peaks.

The candidate fetal R-peaks are evaluated, using a pattern matching technique. For this purpose four FQRS patterns (pFQRS) are defined by an expert cardiologist (Fig. 3). A 20-point length window is centred on each candidate fetal R-peak and the eECG signal in this window is obtained, which is considered as the FQRS. For each window, the maximum and minimum values are obtained and the magnitude is calculated as their difference. The average magnitude for all windows is also calculated. Each candidate fetal R-peak is evaluated as follows: if the normalized cross-correlation between the respective window and one of the pFQRSs is higher than 0.6 and the magnitude is greater than 60% and less than 200% of the average magnitude, then the candidate fetal R-peak is validated, else it is rejected.

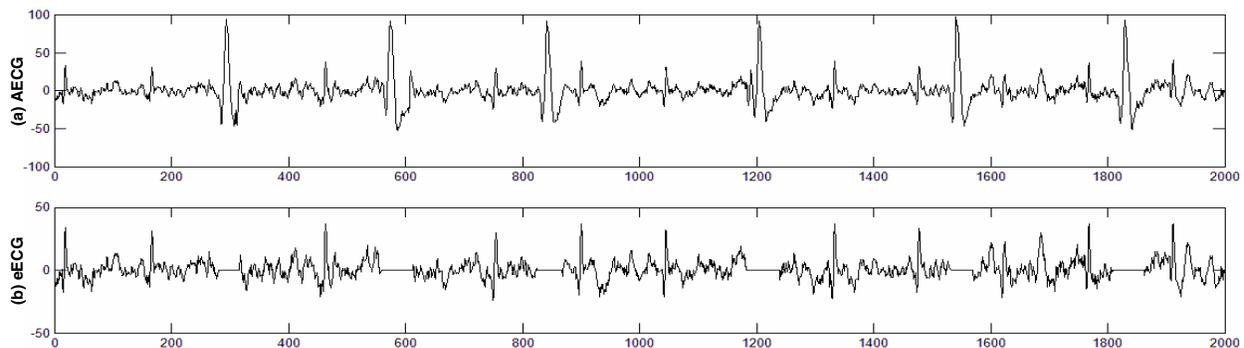


Figure 2. Maternal QRS elimination: (a) AECG, (b) eECG (the MQRSs are eliminated).

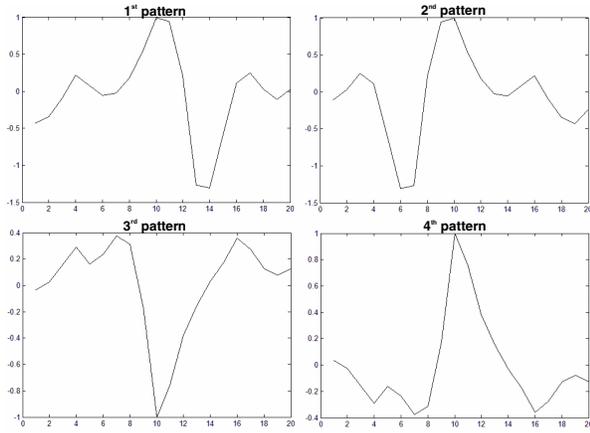


Figure 3. The fetal QRS patterns.

### 2.3. Fetal heart rate extraction

In the final stage, the fetal R-peaks that overlap with the MQRSS are found. For this purpose a histogram-based technique is used. The fetal R-peaks detected from stage 2 are used to accumulate the fRR interval signal and then the histogram of this signal is calculated (Fig. 4). In the histogram, the time corresponding to the first peak (indicated with a circle in Fig. 4) is considered as the “basic” fRR interval length ( $t_{RR}$ ). Then,  $2t_{RR}$  is calculated (indicated in Fig. 4 with a star). All fRR intervals with length  $t \in \left[ 2t_{RR} - \frac{t_{RR}}{2}, 2t_{RR} + \frac{t_{RR}}{2} \right]$  are considered as “double” fRR intervals, i.e. fRR intervals which include one lost fetal R-peak, while all fRR intervals with length  $t > 2t_{RR} + \frac{t_{RR}}{2}$  are considered as “multi” fRR intervals, i.e. fRR intervals that include two or more lost fetal R-peaks. The “double” fRR intervals are split in two fRR intervals, considering a fetal R-peak in the centre of the “double” fRR interval. The “multi” fRR intervals are not corrected and they are considered as high contamination areas of the AECG signal, where the presence of the noise makes the signal absolute. After the application of the stage 3, all fetal R-peaks are detected and they are used to extract the FHR.

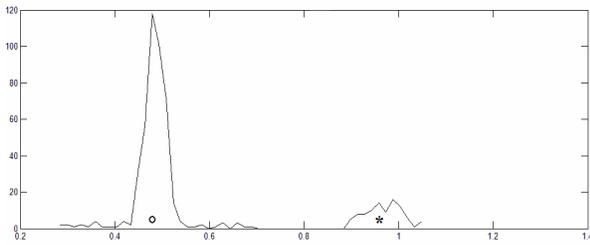


Figure 4. Histogram of the fRR interval signal.

## 3. Results

The database from the University of Nottingham [3] is used for the evaluation of the proposed method. The signals are acquired using four electrodes (three leads and a common) placed on the mother’s abdomen by a low-noise, general-purpose electrophysiological recorder. During the recording, the data were passed through a band-pass filter with high-frequency and low-frequency cutoffs at 100 Hz and 4 Hz respectively, which is the bandwidth of interest for FECG monitoring. The recordings are digitized using a 12-bit analog-to-digital converter. The sampling frequency is set to 300 Hz and the unit’s highest gain (in its FECG configuration) is 7800. The measurements are simultaneously recorded from all three channels. The database consists of 4 long recordings of 15 min, totally including 7596 fetal heart beats. The AECG records are obtained at the 24<sup>th</sup>, 28<sup>th</sup>, 32<sup>nd</sup> and the 34<sup>th</sup> week of gestation.

The results for FQRS detection are evaluated from an expert cardiologist, who calculated three quantitative results: true positive (TP) when a fetal R-peak is correctly detected by the proposed method, false negative (FN) when a fetal R-peak was not detected and false positive (FP) when an artefact is detected as fetal R-peak. Indices of test performance can be derived from these results, such as sensitivity (Se) and positive predictive accuracy (PPA). Also, accuracy (Acc) is calculated, defined as:

$$Acc = \frac{TP}{TP + FP + FN} . \quad (5)$$

Table 1 presents the results obtained from the evaluation of the proposed method. The average Se is 98.39% and the average PPA is 98.93%, while the average Acc is 97.35%. The dataset includes totally 7.263 fetal R-peaks from which 333 were not detected (4.58%), while 198 artefacts are detected as fetal R-peaks (2.72%).

Table 1. Evaluation results.

Record	TP	FP	FN	Se (%)	PPA (%)	Acc (%)
w24	1935	10	82	95.93	99.49	95.46
w28	1915	0	10	99.48	100	99.48
w32	1789	33	30	98.35	98.19	96.60
w34	1624	107	17	98.96	93.82	92.91
<b>Total</b>	<b>7263</b>	<b>198</b>	<b>333</b>	<b>98.39</b>	<b>98.93</b>	<b>97.35</b>

## 4. Discussion

The proposed method addresses several issues related to the FHR extraction: (a) it is based on the analysis of AECG leads. The number of the recorded leads is not of great importance; we use only three leads in the present work and the method can be used with just one lead. This is a major advantage compared to BSS-based methods, since they require a large number of recorded leads to reach a reliable FECG extraction. (b) Thoracic leads are not required, in contrast to several approaches proposed in the literature such as adaptive filtering. (c) Features (a) and (b) enables our method to be embedded into wearable devices and telemedicine applications, where the number of the recorded sources (leads) is limited. (d) The AECG signal varies in time, therefore both time-frequency and wavelets have been utilized. (e) Other methods do not require previous knowledge. Our method requires knowledge about the FQRS shape, which is almost the same for all subjects. (f) Our method automatically extracts the FHR from the AECG leads, in contrast to the BSS-based methods, which results a set of signals and a medical expert must define which of them is the signal of interest (FECG), to be furthermore processed. (g) The proposed method is evaluated using real AECG records, covering a large period of the gestation. Therefore, it is expected to have similar results if it is used under real clinical conditions.

Table 2 presents several methods proposed in the literature for the extraction of the FHR. Due to the fact that there is no benchmark database, each approach is evaluated using different dataset. Therefore, a direct comparison between the results is not feasible. All the methods were validated using real records (no simulated signals were involved) while all leads are placed on the abdomen of the mother (no thoracic leads were used). The proposed method provides comparable results with the other methods. The method proposed by Mooney et al. [6] is not automated; areas of the AECG are initially selected from a user and then the FHR is calculated. The fuzzy-based approach by Azad [12] performed very well with 89% average performance, but there is no reference about the exact number and duration of AECG records that were used for the evaluation. Pieri et al. [3] uses the larger dataset among all the methods presented on Table 2 (400 records of 5-10 min each), but the results are rather poor (65%). The study by Ibahimy et al. [15] is evaluated using the correlation coefficient between the simultaneous FHR measured from Doppler ultrasound and the method (89%).

**Table 2. Comparison of methods.**

Author	Description	Dataset	Acc (%)
Pieri et al., [3]	matched filter	400 records 5-10 min	65
Mooney et al., [6]	adaptive algorithm	several records	100
Azad [12]	fuzzy approach	5 records	89 <sup>1</sup>
Ibahimy et al., [15]	statistical analysis	5 records 20 min	89 <sup>2</sup>
Karvounis et al., [18]	complex wavelets	15 records 1 min	98
<b>Current Work</b>	time-frequency methods	4 long records 15 min	96

<sup>1</sup>Defined as:  $performance = 100 (TP - (FP + FN)) / TP \%$ .

<sup>2</sup>Correlation coefficient between the simultaneous FHR measured from Doppler ultrasound and the method.

A method for the automated extraction of the FHR from the AECG signal has been developed. The method is based on the theory of t-f analysis and CCWT. Real AECG records are incorporated for the evaluation of the method and the presented results indicate very high efficiency, since an overall accuracy of almost 96% is achieved. Both t-f analysis and CCWT are methodologies that have not been used for FHR extraction. The main drawback of the method is the difficulty to extract the fetal R-peaks in noisy background or in cases where the FECG is not distinguishable. The proposed FHR detection method can be further improved, in terms of noise handling. The presence of noise in long duration AECG recordings is unavoidable so proper filtering (either hardware or software) must be utilized. Also, further validation, under real clinical or home-care conditions, with stable or wearable devices, is needed.

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